Confidence Intervals - Practice

Complete the following tasks, discuss in groups.

## Task 1 Heights

Suppose 𝛍 is the average height of a male university student. You measure the heights (in cm) of twenty men, getting data with sample mean .

Suppose that the are drawn from a normal distribution with unknown mean 𝛍 and standard deviation 9 cm.

(a) Construct a 90% z–confidence interval for 𝛍.

| X ~ N(mu, 9^2)  sigma = 9  Z-interval:  sample\_mean +- z\_(1-a/2)\*sigma/sqrt(n)  1 - a = 0.9 => a = 0.1  z\_(1-a/2) = z\_0.95 = 1.64  177 +- 1.64\*9/sqrt(20) |
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(b) How many people in total would you need to measure to bring the width of

the 90% z–confidence interval down to 1 cm?

| 1.64\*9/sqrt(n) <= 1  n >= …  n\* = 218 |
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## Task 2 Soda

Consider a machine that is known to fill soda cans with amounts that follow a normal

distribution with (unknown) mean 𝛍 and unknown standard deviation 𝝈.

We measure the volume of soda in a sample of bottles and obtain the following data (in mL):

*352, 351, 361, 353, 352, 358, 360, 358, 359*

Construct a 95% confidence interval for the mean 𝛍.

| X1...Xn ~ Normal(mu, sigma^2)  mu, sigma are UNKNOWN  t - interval  sample\_mean +- t\_(1-a/2)\*sample\_std/sqrt(n)  sample\_mean = 356  sample\_var = 15.5  sample\_std = sqrt(sample\_var) = sqrt(15.5)  t\_(0.975) = 2.3  **mu = 356 +- 2.3\*sqrt(15.5) /sqrt(9)** |
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## Task 3

In a study on cholesterol levels a sample of 12 men and women was chosen. The plasma cholesterol levels (mmol/L) of the subjects were as follows:

*6.0, 6.4, 7.0, 5.8, 6.0, 5.8, 5.9, 6.7, 6.1, 6.5, 6.3, 5.8*

(a) Estimate the **mean and variance** of the plasma cholesterol levels with a 95% confidence interval.

| CI for mu: t-interval  CI for sigma: chi-squared  (n-1)\*s^2/q\_(1-a/2) < sigma^2 < (n-1)\*s^2/q\_(a/2)  n = 12 => 11 d.f.  q\_(1-a/2) = 21.9  q\_(a/2) = 3.8  s^2 = 0.15  11\*0.15/21.9 < sigma^2 < 11\*0.15/3.8  sqrt(11\*0.15/21.9) < sigma < sqrt(11\*0.15/3.8) |
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(b) What assumptions did you make about the sample in order to make your estimate?

| Only 12 samples -> not enough to apply CLT  Assume that the data is normal  If we had more samples, we could apply CLT if mu, sigma^2 < inf |
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